

# Curvature Quick Reference Sheet

## PHYS 471: Intro to Relativity and Cosmology

- Christoffel symbol

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2}g^{\alpha\beta} \{ \partial_{\mu}g_{\beta\nu} + \partial_{\nu}g_{\mu\beta} - \partial_{\beta}g_{\mu\nu} \}$$

- Riemann curvature tensor

$$R^{\alpha}_{\beta\mu\nu} = \partial_{\mu}\Gamma^{\alpha}_{\beta\nu} - \partial_{\nu}\Gamma^{\alpha}_{\beta\mu} + \Gamma^{\alpha}_{\mu\sigma}\Gamma^{\sigma}_{\beta\nu} - \Gamma^{\alpha}_{\nu\sigma}\Gamma^{\sigma}_{\beta\mu}$$

- All-covariant Riemann curvature tensor

$$R_{\alpha\mu\beta\nu} \equiv g_{\alpha\sigma}R^{\sigma}_{\mu\beta\nu} = g_{\alpha 0}R^0_{\mu\beta\nu} + g_{\alpha 1}R^1_{\mu\beta\nu} + g_{\alpha 2}R^2_{\mu\beta\nu} + g_{\alpha 3}R^3_{\mu\beta\nu}$$

- Number of independent Riemann tensor components

$$\frac{n^2(n^2 - 1)}{12}$$

- Ricci Tensor

$$R_{\mu\nu} \equiv R^{\alpha}_{\mu\alpha\nu} = R^0_{\mu 0\nu} + R^1_{\mu 1\nu} + R^2_{\mu 2\nu} + R^3_{\mu 3\nu}$$

and also

$$R_{\mu\nu} \equiv g^{\alpha\beta}R_{\alpha\mu\beta\nu} = g^{00}R_{0\mu 0\nu} + g^{11}R_{1\mu 1\nu} + g^{22}R_{2\mu 2\nu} + g^{33}R_{3\mu 3\nu}$$

- Ricci Scalar

$$R \equiv g^{\mu\nu}R_{\mu\nu} = g^{00}R_{00} + g^{11}R_{11} + g^{22}R_{22} + g^{33}R_{33}$$

- Symmetries and Identities

Index flip antisymmetry :	$R_{\alpha\mu\beta\nu} = -R_{\mu\alpha\beta\nu} \quad , \quad R_{\alpha\mu\beta\nu} = -R_{\alpha\nu\beta\mu}$
Pair exchange symmetry :	$R_{\alpha\mu\beta\nu} = R_{\beta\nu\alpha\mu}$
Bianchi identity 1 :	$R_{\alpha\beta\mu\nu} + R_{\alpha\mu\nu\beta} + R_{\alpha\nu\beta\mu} = 0$
Bianchi identity 2 :	$D_{\sigma}R_{\alpha\beta\mu\nu} + D_{\mu}R_{\alpha\beta\nu\sigma} + D_{\nu}R_{\alpha\beta\sigma\mu} = 0$
Ricci symmetry :	$R_{\mu\nu} = R_{\nu\mu}$